EX NAVODAYAN FOUNDATION

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B/36/43 46M Brahmanand Colony, Durgakund, Varanasi (UP) 221005

Mob.: 8958143003 Email Id : exnavodayanfoundation@gmail.com

JEE-Main

2nd Revision Minor Test

JEE-Mains Type Test paper

Test Date: 22 Dec, 2024 **M.M:300**

TEST INSTRUCTIONS

1. The test is of **3 hours** duration.
2. The test booklet consists of **75 questions**.
3. The maximum marks are **300**.
4. All questions are compulsory.
5. There are three parts in the questions paper consisting of Physics, Chemistry and Mathematics having **25 questions in each part**.

Each Parts Contains –

- 20 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct. All questions are carrying **+4 marks** for right answer and **–1 mark** for wrong answer.
- 05 questions with answer as **numerical value** all questions are carrying **+4 marks** for right answer and **–1 marks** for wrong answers.

Syllabus: Physics-Electrostatics, Work, Energy and Power, Rotational Motion, Center of Mass, Collision | Chemistry-Chemical thermodynamics, Solutions, Equilibrium | Math-Limit, Continuity and Differentiability, Integral Calculus

Name of the Candidate (in Capital Letter): ____

Registration No. ____

Invigilator Signature

Physics

(Single Correct Choice Type)

This Section contains **20 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. A disc is of mass M and radius r, the moment of inertia of this disc about an axis tangential to its edge and in the plane of the disc is

(a) $\frac{5Mr^2}{4}$ (b) $\frac{5Mr^2}{2}$ (c) $\frac{3Mr^2}{4}$ (d) $\frac{Mr^2}{2}$

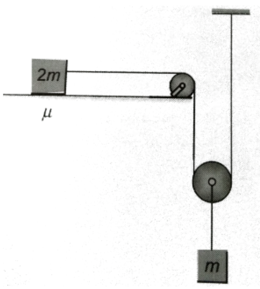
2. A solid sphere of radius R lies on a smooth horizontal surface. It is pulled by a horizontal force acting tangentially from the highest point, find the distance traveled by the sphere during the time it makes one rotation.

(a) $\frac{4\pi R}{5}$ (b) $\frac{4\pi r}{10}$ (c) $\frac{4\pi R}{15}$ (d) $\frac{4\pi r}{25}$

3. A hollow sphere is released from the top of an inclined plane of inclination θ and length L. If a hollow sphere rolls without slipping. What will be its speed when it reaches the bottom?

(a) $\sqrt{\frac{12gL\sin\theta}{5}}$ (b) $\sqrt{\frac{6gL\sin\theta}{5}}$ (c) $\sqrt{\frac{3gL\sin\theta}{5}}$ (d) $\sqrt{\frac{18gL\sin\theta}{5}}$

4. Consider a situation as shown in the figure. The system is released from rest. When the block of mass m has falled a distance L, its speed becomes $\sqrt{\frac{gl}{3}}$. Find the friction coefficient.



(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{10}$ (d) $\frac{1}{2}$

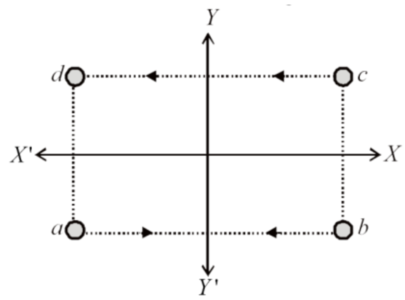
5. Two masses m_1 and m_2 $(m_1 > m_2)$ are connected by massless flexible and inextensible string passed over massless and frictionless pulley. The acceleration of centre of mass is

(a) $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$ (b) $\frac{m_1 - m_2}{m_1 + m_2} g$ (c) $\frac{m_1 + m_2}{m_1 - m_2} g$ (d) zero

6. Two point object of masses 1.5 g and 2.5 g respectively are at a distance of 16 cm a part the centre of gravity is at a distance x from the object of mass 1.5 g where x is

(a) 10 cm (b) 6 cm (c) 13 cm (d) 3 cm

7. Four bodies of equal mass start moving with same speed as shown in the figure. In which of the following combination the centre of mass will remain at origin.

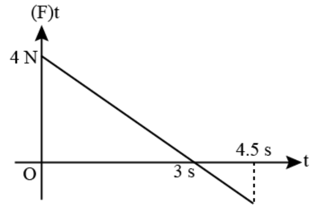


(a) c and d (b) a and b (c) a and c (d) b and d

8. A particle falls from a height h upon a fixed horizontal plane and rebounds. If e is the coefficient of restitution, the total distance travelled before rebounding has stopped is

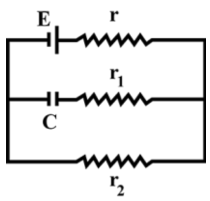
(a) $h\left(\frac{1+e^2}{1-e^2}\right)$ (b) $h\left(\frac{1-e^2}{1+e^2}\right)$ (c) $\frac{h}{2}\left(\frac{1-e^2}{1+e^2}\right)$ (d) $\frac{h}{2}\left(\frac{1+e^2}{1-e^2}\right)$

9. A block of mass 2 kg is free to move along the x-axis. It is at rest and from t = 0 onwards it is subjected to a time-displacement force F(t) in the x-direction the force F(t) varies with t as shown in the figure the kinetic energy of the block after 4.5 seconds is



(a) 4.50 J (b) 7.50 J (c) 5.06 J (d) 14.06 J

10. In the given circuit diagram, when the current reaches steady state in the circuit, the charge o the capacitor of capacitance c will be

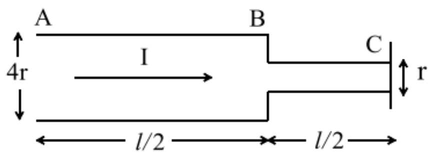


(a) $CE\frac{r_2}{(r_1+r_2)}$ (b) $CE\frac{r_1}{(r_1+r)}$ (c) CE (d) $CE\frac{r_1}{(r_2+r)}$

11. The resistance of the series combination of two resistance is S. When they are joined in Parallel the total resistance is P. If S = n p, then the minimum possible value of n is

(a) 4 (b) 3 (c) 2 (d) 1

12. Consider a cylindrical element as shown in figure current flowing through the element is I and resistivity of material of the cylinder is P. Choose the correct option out of the following.



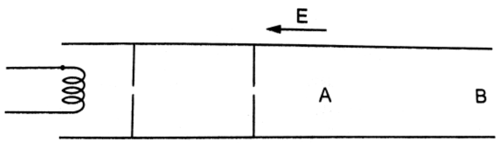
(a) Power loss in second half is four times the power loss in first half

(b) Voltage drop in first half is twice of voltage drop in second half

(c) Current density in both halves is equal

(d) Electric field in both halves is equal

13. Electrons are emitted by a hot filament and are accelerated by an electric field as shown in figure the two stops at the left ensure that the electron beam has a uniform cross-section.



(a) The speed of the electrons is more at B than at A

(b) The electric current is from left to right

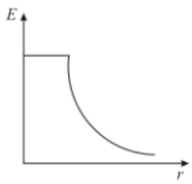
(c) The magnitude of the current is larger at B then at A

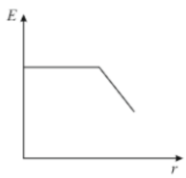
(d) The current density is more at B then at A

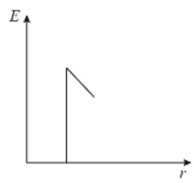
14. Four point charges $q_A = 2\mu_C, q_B = -5\mu_C, q_C = 2\mu_C,$ and $q_D = -5\mu_C$ are located at the corners of a square ABCD of side 10 cm. What is the force on a charge of $1\mu_C$ placed at the center of square?

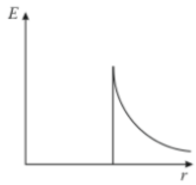
(a) 5 N (b) 2 N (c) zero N (d) 4 N

15. Which one of the following graph shows the variation of electric field strength E with distance r from the center of a hollow conducting sphere.

(a) 

(b) 

(c) 

(d) 

16. An oil drop, carrying six electronic charges and having a mass of $1.6\times10^{-12}g,$ falls with some terminal velocity in a medium. What magnitude of vertical electric field is required to make the drop move upwards with the same speed as it was formerly moving downwards with? Ignore buoyancy

(a) $10^5 N/C$ (b) $10^4 N/C$ (c) $3.3\times10^4 N/C$ (d) $3.3\times10^5 N/C$

17. A shell is fired from a cannon with velocity v m/sec at an angle θ with the horizontal direction. At the highest point in its path it explodes into pieces of equal mass one of the pieces retraces its path to the cannon and the speed in m/sec of the other piece immediately after the explosion is

(a) $3v\cos\theta$ (b) $2v\cos\theta$ (c) $\frac{3}{2}v\cos\theta$ (d) $\frac{\sqrt{3}}{2}v\cos\theta$

18. When two bodies collide elastically then.

(a) kinetic energy of the system alone is conserved

(b) only momentum is conserved

(c) Both energy and momentum are conserved

(d) Neither energy nor momentum is conserved

19. A solid sphere, a hallow sphere and a ring are released from top of an inclined plane (frictionless) so that they slide down the plane, then maximum acceleration down the plane is for (no rolling)

(a) solid sphere (b) Hollow sphere (c) Ring (d) All same

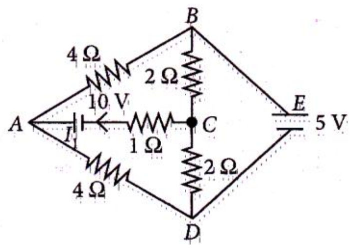
20. A uniform chain of mass m and length L overhangs a rough horizontal table with its two third part on the table. Find the work done by the friction during the period the chain slips off the table.

(a) $-\frac{\mu mgL}{9}$ (b) $-\frac{4\mu mgL}{9}$ (c) $-\frac{2\mu mgL}{9}$ (d) $-\frac{2\mu mgL}{11}$

(Integer Type Questions)

This Section contains **5 Questions.** The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

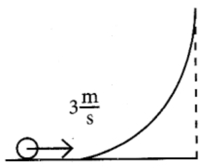
1. The current flowing through the 1 Ω resistor is $\frac{n}{10}$A. The value of n is ___________.



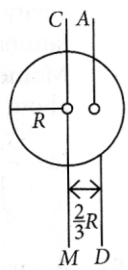
2. At the centre of a half ring of radius R = 10 cm and linear charge density $4\,nC\,m^{-1}$, the potential is $x\pi V$. The value of x is ______________.

3. A body of mass 1 kg collides head on elastically with a stationary body of mass 3 kg. After collision, the smaller body reverses its direction of motion and moves with a speed of 2 m/s. The initial speed of the smaller body before collision is __________ ms^{-1}.

4. A hollow spherical ball of uniform density rolls up a curved surface with an initial velocity 3 m/s (as shown in figure). Maximum height with respect to the initial position covered by it will be ____cm.



5. I_{CM} is the moment of inertia of a circular disc about an axis (CM) passing throigh its center and perpendicular to the plane of disc. I_{AB} is it's moment of inertia about an axis AB perpendicular o plane and parallel to axis CM at a distance $\frac{2}{3}R$ from center. Where R is the radius of the disc. The ratio of I_{AB} and I_{CM} is x : 9. The value of x is _________.



Chemistry

(Single Correct Choice Type)

This Section contains **20 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. Which of the following statements is not true regarding vapour pressure of solvent (p°) and that of the solution (P_s)containing non-volatile solute?

   (a) Both p° and p_s increase on increasing temperature

   (b) $(p° - p_s)$ increases on increasing temperature

   (c) $p_s = p° \times$ mole fraction of solvent

   (d) $\frac{(p° - p_s)}{p°}$ decreasing on increasing temperature

2. Vapour pressure (in torr) of an ideal solution of two liquids A and B is given by : $P = 52X_A + 114$ where X_A is the mole fraction of A in the mixture. The vapour pressure (in torr) of equimolar mixture of the two liquids will be

   (a) 166 (b) 83 (c) 140 (d) 280

3. π_1, π_2, π_3 and π_4 atm are the osmotic pressure of 5% (mass/volume) solutions of urea, fructose, sucrose and KCl respectively at certain temperature. The correct order of their magnitudes is

   (a) $\pi_1 > \pi_4 > \pi_2 > \pi_3$ (b) $\pi_1 < \pi_4 < \pi_2 < \pi_3$ (c) $\pi_4 > \pi_1 > \pi_2 > \pi_3$ (d) $\pi_4 > \pi_1 > \pi_3 > \pi_2$

4. Henery's law constant K of CO_2 in water at 25°C is $3.0 \times 10^{-2} moL^{-1}atm^{-1}$. Calculate the mass of CO_2 present in 100 L of soft drink bottled with a partial pressure of CO_2 of 4 atm at the same temperature

   (a) 5.28g (b) 12.0g (c) 428g (d) 528g

5. The osmosis is a process in which –

   (a) Solute molecules move from solution of lower concentration to higher concentration through semipermeable membrane

   (b) Solvent molecules move from solution of lower concentration to higher concentration through semipermeable membrane

   (c) Solvent molecules move from solution of higher concentration to lower concentration through semipermeable membrane

   (d) None of these

6. 7.6 gm KBr in 1250 ml solution was found to have an osmotic pressure of 1.804 atm at 26°C. Calculate degree of ionization and Van't Hoff factor.

(a) 43.4% (b) 45.4% (c) 4.34% (d) 4.45%

7. The exothermic formation of a CIF_3 is represented by the equation

$Cl_2(g) + 3F_2(g) \rightleftharpoons 2ClF_3(g);\ \Delta H = -329 kJ$

Which of the following will increase the quantity of ClF_3 in an equilibrium mixture of Cl_2, F_2 and ClF_3 ?

(a) Adding F_2 (b) Increasing the volume of the container

(c) Removing Cl_2 (d) Increasing the temperature.

8. Phosphorus pentachloride dissociates as follows, in a closed reaction vessel

$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$

If total pressure at equilibrium of the reaction mixture is P and degree of dissociation of PCl_5 is x, the partial pressure of PCl_3 will be

(a) $\left(\frac{x}{x-1}\right)P$ (b) $\left(\frac{x}{1-x}\right)P$ (c) $\left(\frac{x}{x+1}\right)P$ (d) $\left(\frac{2x}{1-x}\right)P$

9. The pK_a of a weak acid, HA, is 4.80. the pK_p of a weak base, BOH, is 4.78. The pH of an aqueous solution of the corresponding salt, BA, will be :

(a) 9.22 (b) 9.58 (c) 4.79 (d) 7.01

10. Which of the following solutions is alkaline ?

(a) KCl solution (b) CH_3COONH_4 solution

(c) $FeCl_3$ solution (b) KCN solution

11. The molar solubility (in mol L^{-1}) of a sparingly soluble salt MX_4 is s. The corresponding solubility product is K_{sp}. S is given in terms of K_{sp} by the relation .

(a) $s = (K_{sp}/128)^{1/4}$ (b) $s = (128K_{sp})^{1/4}$ (c) $s = (256K_{sp})^{1/5}$ (d) $s = (K_{sp}/256)^{1/5}$

12. Solid $Ba(NO_3)_2$ is gradually dissolved in a 1.0×10^{-4} M Na_2CO_3 solution. At what concentration of Ba^{2+} will a precipitate begin to form ?

(Given : $K_{sp} = 5.1 \times 10^{-9} M^2$ for $BaCO_3$)

(a) 8.1×10^{-8} M (b) 8.1×10^{7} M (c) 4.1×10^{5} M (d) 5.1×10^{-5} M

13. For pure water ,

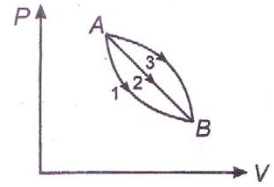
(a) pH increases with increase in temperature

(b) pH decreases with increase in temperature

(c) pH = 7 and is independent of temperature

(d) pH increases at low temperatures but decreases at high temperatures

14. A given mass of gas expands from the state A to the state B by three paths 1, 2 and 3 as shown in the figure. If w_1, w_2 and w_3 respectively be the work done by the gas along three paths then:



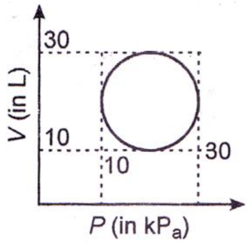
(a) $w_1 > w_2 > w_3$

(b) $w_1 < w_2 < w_3$

(c) $w_1 = w_2 = w_3$

(d) $w_2 < w_3 < w_1$

15. Heat absorbed by a system in going through a cyclic process shown in figure is :



(a) $10^7\ \pi J$

(b) $10^6\ \pi J$

(c) $10^2\ \pi J$

(d) $10^4\ \pi J$

16. The enthalpy changes for tow reactions are given by the equations :

$$2Cr(s) + 1\frac{1}{2}O_2(g) \longrightarrow Cr_2O_3(s); \quad \Delta H = -1130 kJ$$

$$C(s) + \frac{1}{2}O_2(g) \longrightarrow CO(g); \quad \Delta H = -110 kJ$$

What is the enthalpy change, in kJ, for the reaction?

$$3C(s) + Cr_2O_3(s) \longrightarrow 2Cr(s) + 3CO(g)$$

(a) -1460 kJ (b) -800 kJ (c) + 800 kJ (d) + 1020 kJ (e) + 1460 kJ

17. One gram mole of graphite and diamond were burnt to form CO_2 gas.

$$C_{(graphite)} + O_2(g) \longrightarrow CO_2(g); \quad \Delta H^0 = -399.5 \text{ kJ}$$

$$C_{(diamond)} + O_2(g) \longrightarrow CO_2(g); \quad \Delta H^0 = -395.4 \text{ kJ}$$

(a) Graphite is more stable than diamond (b) Diamond is more stable than graphite

(c) Graphite has greater affinity with oxygen (d) Diamond has greater affinity with oxygen

18. In C_2H_4, energy of formation of (C=C) and (C–C) are –145 kJ/mol and –80 kJ/mol respectively. What is the enthalpy change which ethylene polymerises to form polythene?

(a) + 650 kJ/mol (b) + 65 kJ/mol (c) –650 kJ/mol^{-1} (d) –65 kJ mol^{-1}

19. In Mayer's relation, $C_P - C_V = R$

'R' stands for :

(a) translational kinetic energy of 1 mol gas

(b) rotational kinetic energy of 1 mol gas

(c) vibrational kinetic energy of 1 mol gas

(d) work done to increase the temperature of 1 mol gas by one degree

20. Given the following date :

Substance	$\Delta H^0(\text{kJ/mol})$	$S^0(\text{J/mol K})$	$\Delta G^0(\text{kJ/mol})$
FeO(s)	-266.3	57.49	-245,12
C(Graphite)	0	5,74	0
Fe(s)	0	27.28	0
CO(g)	-110.5	197.6	-137.15

Determine at what temperature the following reaction is spontaneous ?

$$FeO(s) + C(Graphite) \rightarrow Fe(s) + CO(g)$$

(a) 298 K (b) 668 K

(c) 966 K (d) ΔG^0 is +ve, hence the reaction will never be spontaneous

(Integer Type Questions)

This Section contains 5 **Questions.** The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

1. A beaker containing 18g of glucose in 100g water and another containing 18g of urea in 100g water are placed under at bell jar and air is removed. After a course of time when equilibrium reaches, how much water in gram will be transferred from one beaker to the other?

2. If the concentration of OH^- ions in the reaction

$Fe(OH)_3(s) \rightleftharpoons Fe^{3+}(aq) + 3OH^-(aq)$ is decreased by $\frac{1}{4}$ times, then equilibrium concentration of Fe^{3+} will increase by ……. times

3. A 200 mL 5×10^{-5} M HCl solution, is mixed with another 300 mL 5×10^{-5} M NaOH solution at 25°C. Assuming temperature to be constant, pH of the resulting solution is ________

4. The S—S bond energy in KJ is if $\Delta H_f^o(E_t - S - E_t) = -147$ kJ/mol. $\Delta H_f^o(E_t - S - S - E_t) = -202$ kJ/mol and ΔH_f^o S(g) = +233 kJ/mol:

5. One mole of an ideal gas expands reversibly and adiabatically from a temperature of 27^0C. If the work done during the process is 3 kJ, then final temperature of the gas in Kelvin is :
($C_v = 20 J/K$)

Mathematics

(Single Correct Choice Type)

This Section contains **20 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

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1. Let $n \geq 2$ be a natural number and $0 < \alpha < \frac{\pi}{2}$. Then $\int \frac{(\sin^n\theta - \sin\theta)^{\frac{1}{n}}\cos\theta}{\sin^{n+1}\theta}d\theta$ is equal to :

(a) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$ (b) $\frac{n}{n^2+1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

(c) $\frac{n}{n^2-1}\left(1+\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$ (d) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n+1}\theta}\right)^{\frac{n+1}{n}}+C$

(where C is constant of integration)

2. For a real number y , let [y] denotes the greatest integer less than or equal to y: Then the function $f(x) = \frac{\tan(\pi[x-\pi])}{1+[x]^2}$ is

(a) discontinuous at some x
(b) continuous at all x, but the derivative f′(x) does not exist for some x
(c) f′(x) exists for all x , but the second derivative f′(x) does not exist for some x
(d) f′(x) exists for all x

3. $\lim_{x\to\frac{\pi}{2}} \frac{\left[1-\tan\left(\frac{x}{2}\right)\right][1-\sin x]}{\left[1+\tan\left(\frac{x}{2}\right)\right][\pi-2x]^3}$ is

(a) ∞ (b) $\frac{1}{8}$ (c) 0 (d) $\frac{1}{32}$

4. Let $f(x)=\int \frac{2x}{(x^2+1)(x^2+3)}dx.$. If $f(3)=\frac{1}{2}(\log_e 5-\log_e 6)$, then $f(4)$ is equal to

(a) $\frac{1}{2}\log_e 17-\log_e 19$ (b) $\log_e 17-\log_e 18$

(c) $\frac{1}{2}(\log_e 19-\log_e 17)$ (d) $\log_e 19-\log_e 20$

5. $\lim_{x\to 0}\frac{e^{2|\sin x|}-2|\sin x|-1}{x^2}$

(a) is equal to 1 (b) does not exist (c) is equal to -1 (d) is equal to 2

6. Let $\int_{\alpha}^{\log_e 4}\frac{dx}{\sqrt{e^x-1}}=\frac{\pi}{6}$. Then e^{α} and $e^{-\alpha}$ are the roots of the equation:

(a) $2x^2-5x+2=0$ (b) $x^2-2x-8=0$ (c) $2x^2-5x-2=0$ (d) $x^2+2x-8=0$

7. Let x = 2 be a root of the equation $x^2+px+q=0$ and

$$f(x)\begin{cases}\frac{1-\cos(x^2-4px+q^2+8q+16)}{(x-2p)^4} & ,x\neq 2p\\ 0 & ,x=2p\end{cases}$$

Then $\lim_{x\to 2p^+}[f(x)]$ where [.] denotes greatest integer function, is

(a) 2 (b)1 (c) 0 (d) -1

8. Let $f(x)$ be a polynomial of degree 4 having extreme values at x =1 and x =2. If $\lim_{x\to 0}\left(\frac{f(x)}{x^2}+1\right)=3$ then $f(-1)$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{9}{2}$

9. The integral $\int \frac{\sin^2 x\cos^2 x}{(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x)^2}dx$ is equal to :

(a) $\frac{-1}{3(1+\tan^3 x)}+C$ (b) $\frac{1}{1+\cot^3 x}+C$

(c) $\frac{-1}{1+\cot^3 x}+C$ (d) $\frac{1}{3(1+\tan^3 x)}+C$

(where C is constant of integration)

10. Let g(x) be a linear function and $f(x)=\begin{cases}g(x) & ,x\leq 0\\ \left(\frac{1+x}{2+x}\right)^{\frac{1}{x}} & ,x>0\end{cases}$, is continuous at x = 0. If $f'(1)=f(-1)$, then the value g(3) is

(a) $\frac{1}{3}\log_e\left(\frac{4}{9e^{\frac{1}{3}}}\right)$ (b) $\log_e\left(\frac{4}{9e^{\frac{1}{3}}}\right)$ (c) $\frac{1}{3}\log_e\left(\frac{4}{9}\right)+1$ (d) $\log_e\left(\frac{4}{9}\right)-1$

11. If the value of the integral $\int_{-1}^{1} \frac{\cos \alpha x}{1+3^x} dx$ is $\frac{2}{\pi}$. Then , a value of α is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

12. Let $f(x)=[x^2-x]+|-x+[x]|$, where $x \in R$ and [t] denotes the greatest integer less than or equal to t. Then, f is

(a) continuous at x = 0 , but not continuous at x = 1

(b) continuous at x = 0 and x =1

(c) not continuous at x= 0 and x = 1

(d) continuous at x = 1, but not continuous at x = 0

13. $\lim_{x \to -0} \left(\frac{x - \sin x}{x} \right) \sin\left(\frac{1}{x} \right)$

(a) equals 1 (b) equals 0 (c) does not exist (d) equals -1

14. If the function $f(x)=\begin{cases} \frac{72^x - 9^x - 8^x + 1}{\sqrt{2} - \sqrt{1+\cos x}} & , x \neq 0 \\ a \log_2 2 \log_e 3 & , x = 0 \end{cases}$ is continuous at x = 0, then the value of a^2 is equal to

(a) 968 (b) 1152 (c) 746 (d) 1250

15. Suppose for a differentiable function h,h(0) = 0, h(1) =1 and h′(0) =h′(1) =2. If g(x) = h(e^x)$e^{h(x)}$, then g′(0) is equal to :

(a) 5 (b) 3 (c) 8 (d) 4

16. Which of the following statements is incorrect for the function $g(\alpha)$ for $\alpha \in R$ such that

$$g(\alpha)=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin^{\alpha} x}{\cos^{\alpha} x + \sin^{\alpha} x} dx$$

(a) $g(\alpha)$ is a strictly increasing function (b) $g(\alpha)$ has an inflection point at $\alpha = -2$

(c) $g(\alpha)$ is a odd decreasing function (d) $g(\alpha)$ is an even function

17. Let $k \in R$. If $\lim_{x \to 0^+} (\sin(\sin kx) + \cos x + x)^{\frac{2}{x}} = e^6$, then the value of k is

(a) 1 (b) 2 (c) 3 (d) 4

18. The temperature T(t) of a body at time t = 0 is 160^o F and it decreases continuously as per the differential equation $\frac{dT}{dt} = -K(T-80)$, where K is positive constant. If T(15) =120^o F, then T(45) is equal to

(a) $85^o F$ (b) $95^o F$ (c) $90^o F$ (d) $80^o F$

19. If (a,b) be the orthocenter of the triangle whose vertices are (1,2) , (2,3) and (3,1) and $I_1 = \int_a^b x \sin\left(4x - x^2\right) dx$, $I_2 = \int_a^b \sin(4x - x^2)\, dx$, then $36 \frac{I_1}{I_2}$ is equal to :

(a) 72 (b) 88 (c) 80 (d) 66

20. The area of the region enclosed by the curve $f(x) = \max\{\sin x, \cos x\}, -\pi \le x \le \pi$ and the x-axis is

(a) $2(\sqrt{2}+1)$ (b) $2\sqrt{2}(\sqrt{2}+1)$ (c) $4\left(\sqrt{2}\right)$ (d) 4

(Integer Type Questions)

This Section contains **5 Questions.** The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

1. Let A be the area bounded by the curve $y = x|x-3|$, the x-axis and the ordinates x = -1 and x =2. Then 12 A is equal to.

2. Let m and n be two positive integers greater than 1. If $\lim_{\alpha \to 0} \left(\frac{e^{\cos(\alpha^n)} - e}{\alpha^m} \right) = -\left(\frac{e}{2} \right)$ then the value of $\frac{m}{n}$ is

3. If $I = \frac{2}{\pi} \int_{-\pi/4}^{\pi/4} \frac{dx}{(1+e^{\sin x})(2-\cos 2x)}$, then $27 I^2$ equals ____.

4. Let k and m be positive real numbers such that the function $f(x) = \begin{cases} 3x^2 + k\sqrt{x+1} & ,0 < x < 1 \\ mx^2 + k^2 & ,x \ge 1 \end{cases}$ is differentiable for all x > 0. Then $\frac{8f'(8)}{f'\left(\frac{1}{8}\right)}$ is equal to ____.

5. If $\int \frac{1}{\sqrt[5]{(x-1)^4 (x+3)^6}} dx = A \left(\frac{\alpha x - 1}{\beta x + 3} \right)^B + C$, where C is the constant of integration , then the value of $\alpha + \beta + 20AB$ is ___.

Date- 22-12-2024 **Answer – key**

2nd Revision Minor JEE-Main Test (Main Type)

Physics

1. a
2. a
3. b
4. b
5. a
6. a
7. c
8. a
9. c
10. a
11. a
12. a
13. a
14. c
15. d
16. c
17. a
18. c
19. d
20. c

Integer

1. 25
2. 36
3. 4
4. 75
5. 17

Chemistry

1. d
2. c
3. c
4. d
5. b
6. a
7. a
8. c
9. d
10. d
11. d
12. d
13. b
14. b
15. c
16. c
17. a
18. b
19. d
20. c

Integer

1. 50
2. 64
3. 6
4. 278
5. 150

Maths

1. a
2. d
3. d
4. a
5. d
6. a
7. c
8. d
9. a
10. b
11. b
12. d
13. b
14. b
15. d
16. a
17. b
18. c
19. a
20. d

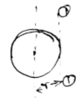
Integer

1. 62
2. 2
3. 4
4. 309
5. 7

2nd Revision Minor JEE-Main Test (Main Type)

PHYSICS

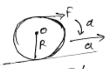
1. (a)



$I_{1-1} = (I_{c.m})_{dia} + M^2 r$

$= \frac{1}{4} mr^2 + mr^2 = \frac{5}{4} mr^2$

2. (a)



$F = ma \Rightarrow a = F/m$

$\tau_0 = FR = I_0 \alpha = \frac{2}{5} mR^2 \alpha \Rightarrow \alpha = \frac{5F}{2mR}$

Angular motion $\theta = \frac{1}{2} \alpha t^2$

Linear motion $s = \frac{1}{2} at^2$

$\frac{s}{\theta} = \frac{a}{\alpha}$

$\frac{s}{2\pi} = \frac{2R}{5} \Rightarrow \boxed{s = \frac{4\pi R}{5}}$

3. (b)

$v = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}} = \sqrt{\frac{2gL \sin\theta}{1 + \frac{2}{3}}} = \sqrt{\frac{6}{5} gL \sin\theta}$

4. (b)

When the block of mass m has descended a distance L, distance traveled by the block of mass 2m is 2L. If the speed of m is v, speed of 2m will be 2v.

Work done by friction $= \mu \times 2mg \times 2L = 4\mu mgL$

Loss in P.E. = gain in K.E. + work done against friction

$mgL = \frac{1}{2}mv^2 + \frac{1}{2} \times 2m(2v)^2 + 4\mu mgL$

$= \frac{9}{2}mv^2 + 4\mu mgL$

$= \frac{9}{2} m \times \frac{gL}{9} + 4\mu mgL$

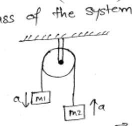
$\frac{mgL}{2} = 4\mu mgL \Rightarrow \underline{\mu = 1/8}$

5. (a)

Acceleration of each mass $= a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$

Now acceleration of centre of mass of the system

$A_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$



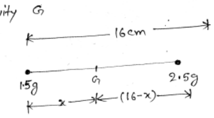
as both masses move with same acceleration but in opposite direction so $\vec{a}_1 = -\vec{a}_2 = a$ (let)

$\therefore A_{cm} = \frac{m_1 a - m_2 a}{m_1 + m_2}$

$= \left(\frac{m_1 - m_2}{m_1 + m_2}\right) \times \left(\frac{m_1 - m_2}{m_1 + m_2}\right) \times g = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 \times g$

6. (a)

Taking the moment of forces about center of gravity G



$(1.5)gx = 2.5g(16 - x) \Rightarrow 3x = 80 - 5x$

$8x = 80 \Rightarrow x = \underline{10\text{ cm}}$

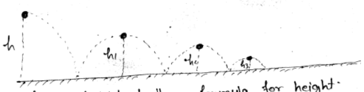
7. (c)

Center of mass lies always on the line that joins the two particals

for the combination cd and ab is line does not pass trough the origin.

For combination bd, initially it passes through the origin but for combination ac it will always pass through origin. So we can say that centere of mass of this combination will remain at origin.

8. (a)



Partical falls from height h then formula for height covered by in nth rebound is given by

$h_n = he^{2n}$

where e = cofficient of restitution, n = No of rebound

Total distance travelled by particle before rebounding has stopped.

$H = h + 2h_1 + 2h_2 + 2h_3 + 2h_4 + \cdots$

$h + 2he^2 + 2he^4 + 2he^6 + 2he^8 + \cdots$

$h + 2h(e^2 + e^4 + e^6 + e^8 + \cdots)$

$= h + 2h\left[\frac{e^2}{1 - e^2}\right] = h\left[1 + \frac{2e^2}{1 - e^2}\right]$

$\boxed{= h\left(\frac{1 + e^2}{1 - e^2}\right)}$

9. (c)

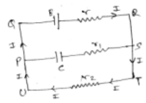
$\int F\,dt = \Delta p$

$\Rightarrow \frac{1}{2} \times 4 \times 3 - \frac{1}{2} \times 1.5 \times 2 = P_f - 0 = P_f = 6 - 1.5 = \frac{9}{2}$

$K.E = \frac{p^2}{2m} = \frac{81}{4 \times 2 \times 2} = K.E = 5.06\text{ J}.$

10. (a)

At steady state no current flows throuh capacitor C. thus the current through the network will be as shown in fig then



$I = \frac{E}{r + r_2}$

Potential difference across ~~and~~ p and S

$V_S - V_P = Ir_2$ charge on the capacitor

$Q = C(V_S - V_P) = CIr_2 = C \times \frac{E}{r + r_2} r_2 = \frac{CEr_2}{r + r_2}$

11. (a)

Let R_1 and R_2 be the two given resistance.
As per question
$S = R_1 + R_2$, $P = \frac{R_1 R_2}{R_1 + R_2}$; $S = nP$

∴ $R_1 + R_2 = \frac{nR_1R_2}{R_1+R_2}$ or $R_1^2 + R_2^2 + 2R_1R_2 = nR_1R_2$

or $(R_1 - R_2)^2 + 4R_1R_2 = nR_1R_2$

or $(R_1 - R_2)^2 = R_1R_2(n-4)$

If $R_1 = R_2$ then $(n-4) = 0$ or $n = 4$

12. (a)

$R_1 = \frac{\rho(l/2)}{\pi(2r)^2} = \frac{\rho l}{8\pi r^2}$

$R_2 = \frac{\rho(l/2)}{\pi r^2} = \frac{\rho l}{2\pi r^2}$

∴ $\frac{R_1}{R_2} = \frac{1}{4}$

Power loss, $P = I^2R$ or $P \propto R$

∴ $\frac{P_1}{P_2} = \frac{R_1}{R_2} = \frac{1}{4}$ or $P_2 = 4P_1$

Voltage drop $V = IR$ or $V \propto R$

∴ $\frac{V_1}{V_2} = \frac{R_1}{R_2} = \frac{1}{4}$ or $V_2 = 4V_1$

current density $J = \frac{I}{A} = \frac{I}{\pi r^2}$

or $J \propto \frac{1}{r^2}$

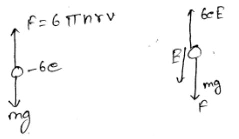
∴ $\frac{J_1}{J_2} = \frac{r^2}{(2r)^2} = \frac{1}{4}$

$J_2 = 4J_1$

13. (a)
14. (c)
15. (d)
Inside a hollow conductor electric field is zero at surface it is maximum and decreases
16. (c)

For the first case: $F = mg$

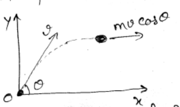
For the second case: $F + mg = 6eE = 2mg = 6eE$



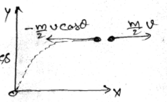
$E \Rightarrow \frac{mg}{3e} \Rightarrow \frac{1.6\times10^{-15}\times10}{3\times1.6\times10^{-19}} = 3.3\times10^{4}\ N/C$

17. (a)

Shell is fired with velocity v at an angle with the horizontal.
So its velocity at the highest point = horizontal component of velocity = $v\cos\theta$
So momentum of shell before explosion = $mv\cos\theta$



When it breaks into two equal pieces one piece retraces its path to the cannon, then other moves with velocity υ



So momentum of two pieces after explosion
$= \frac{m}{2}(-v\cos\theta) + \frac{m}{2}\upsilon$
By the law of conservation of momentum.
$mv\cos\theta = -\frac{m}{2}v\cos\theta + \frac{m}{2}\upsilon \Rightarrow \upsilon = 3v\cos\theta$

18. (c)

19. (d)

as the inclined plane is frictionless before all the bodies will slide down along the inclined plane with same acceleration $g\sin\theta$

20. (c)

Let at some instant the hanging length is x.
Friction acting on the chain

$f = \mu\frac{m}{L}(L-x)g$

Small work done by friction during displacement dx

$dW = -f\,dx$

Total work done from $x = \frac{L}{3}$ to $x = L$

$W = -\int_{L/3}^{L}\frac{\mu mg}{L}(L-x)dx$

$= -\frac{\mu mg}{L}\left|Lx - \frac{x^2}{2}\right|_{L/3}^{L}$

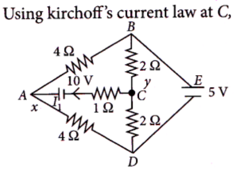
$= -\frac{\mu mg}{L}\left[\left\{L\times L - \frac{L^2}{2}\right\} - \left\{L\times\frac{L}{3} - \frac{1}{2}\left(\frac{L}{3}\right)^2\right\}\right]$

$= -\frac{\mu mg}{L}\cdot\frac{4L^2}{18} = -\frac{2\mu mgL}{9}$

Integer Type

1. (25)

Using kirchoff's current law at C,



Let potential at point C be y.

$\frac{y-5}{2} + \frac{y-0}{2} + \frac{y-x+10}{1} = 0$

$4y - 2x + 15 = 0$ ...(i)

Let potential at point A be x. Now use KCL at A,

$\frac{x-5}{4} + \frac{x-0}{4} + \frac{x-10-y}{1} = 0$

$6x - 4y - 45 = 0$ ...(ii)

Using equations (i) and (ii) we have,

$x = 15/2, y = 0$

$I_1 = \frac{y-x+10}{1} = 0 - 7.5 + 10 = 2.5\,A$

So, $n = 25$.

2. (36)

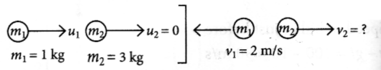
$R = 10$ cm, $\lambda = 4$ nC/m $= 4\times10^{-9}$ C/m

$V = \frac{kQ}{R} = \frac{k\lambda\pi R}{R}$

∴ $V = k\lambda\pi = 9\times10^{9}\times4\times10^{-9}\times\pi V$

$V = 36\,\pi V$

3. (4)



Given elastic collision, $e = 1$ (using conservation of momentum)

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \Rightarrow 1u_1 + 0 = -1\times2 + 3v_2$

$u_1 = 3v_2 - 2$ ...(i)

$e = 1 = \frac{v_2-(-2)}{u_1}$; $u_1 = v_2 + 2$ ...(ii)

From (i) and (ii); $3v_2 - 2 = v_2 + 2$

$v_2 = 2$ m/s and from (ii); $u_1 = 2 + 2 = 4$ m/s

4. (75)

From energy conservation,

$U_i + K_i = U_f + K_f$

$0 + \left(\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\right) = mgh + 0$

$\Rightarrow \frac{1}{2}\times(3)^2 + \frac{1}{2}\times 9\times\frac{2}{3} = 10\times h \Rightarrow \frac{9}{2} + 3 = 10h$

$\Rightarrow \frac{9}{2} + 3 = 10h \Rightarrow \frac{15}{2} = 10h \Rightarrow h = \frac{15}{20} = 0.75\text{ m} = 75\text{ cm}$

5. (17)

$I_{cm} = \frac{1}{2}MR^2$

$I_{AB} = I_{cm} + Mh^2$ (By parallel axis theorem) $\left(h = \frac{2}{3}R\right)$

$I_{AB} = \frac{1}{2}MR^2 + M\left(\frac{2R}{3}\right)^2$

$I_{AB} = \frac{1}{2}MR^2 + \frac{M\times 4R^2}{9} = \frac{17MR^2}{18}$

$\frac{I_{AB}}{I_{cm}} = \frac{17\times 2}{18\times 1} = \frac{17}{9}$, so $x = 17$.

Mathematics

1. Sol.(a)

Let, $I = \int \frac{(\sin^n\theta - \sin\theta)^{\frac{1}{n}}\cos\theta}{\sin^{n+1}\theta}d\theta$

Let $\sin\theta = u \Rightarrow \cos\theta\, d\theta = du$

$\therefore\ I = \int \frac{(u^n - u)^{\frac{1}{n}}}{u^{n+1}}du$

$= \int \frac{\left(1 - \frac{1}{u^{n-1}}\right)^{\frac{1}{n}}}{u^n}du = \int u^{-n}(1 - u^{1-n})^{\frac{1}{n}}du$

Let $1 - u^{1-n} = v$

$\Rightarrow -(1-n)u^{-n}\,du = dv \Rightarrow u^{-n}\,du = \frac{dv}{n-1}$

$\therefore\ I = \int v^{\frac{1}{n}}\cdot\frac{dv}{n-1} = \frac{1}{n-1}\cdot\frac{v^{\frac{1}{n}+1}}{\frac{1}{n}+1}$

$= \frac{n}{n^2-1}v^{\frac{n+1}{n}} + C = \frac{n}{n^2-1}\left(1 - \frac{1}{u^{n-1}}\right)^{\frac{n+1}{n}} + C$

$= \frac{n}{n^2-1}\left(1 - \frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$

2. Sol.(d)

Given : $f(x) = \frac{\tan(\pi[x-\pi])}{1+[x]^2}$

Clearly $[x - \pi]$ is an integer whatever be the value of x.

$\therefore$ $\pi[x-\pi]$ is an integral multiple of π.

Consequently $\tan(\pi[x-\pi]) = 0, \forall x$.

Also $1 + [x]^2 \neq 0$ for any x.

$\therefore$ $f(x) = 0$.

Hence, $f(x)$ is constant function and therefore [illegible] continuous and differentiable any number of times. [illegible] $f'(x), f''(x), f'''(x),\ldots$ all exist for every x, their value [illegible] at every point x. Hence, out of all the alternatives [illegible] is correct.

3. Sol.(d)

$\lim_{x\to\frac{\pi}{2}} \frac{\tan\left(\frac{\pi}{4} - \frac{x}{2}\right).(1-\sin x)}{(\pi - 2x)^3}$

Let $x = \frac{\pi}{2} + y;\ y \to 0$

$= \lim_{y\to 0}\frac{\tan\left(-\frac{y}{2}\right).(1-\cos y)}{(-2y)^3} = \lim_{y\to 0}\frac{-\tan\frac{y}{2}2\sin^2\frac{y}{2}}{(-8).\frac{y^3}{8}.8}$

$\left[\because 1 - \cos\theta = 2\sin^2\frac{\theta}{2}\right]$

$= \lim_{y\to 0}\frac{1}{32}\frac{\tan\frac{y}{2}}{\left(\frac{y}{2}\right)}.\left[\frac{\sin y/2}{y/2}\right]^2 = \frac{1}{32}$

$\left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = \lim_{\theta\to 0}\frac{\tan\theta}{\theta} = 1\right]$

4. Let(a)

We have $f(x) = \int \frac{2x}{(x^2+1)(x^2+3)}\cdot dx$

Put $x^2 = t$

$\int \frac{dt}{(t+1)(t+3)} = \frac{1}{2}\int\left(\frac{1}{t+1} - \frac{1}{t+3}\right)dt$

$f(x) = \frac{1}{2}\ln\left(\frac{x^2+1}{x^2+3}\right) + C$

Given, $f(3) = \frac{1}{2}(\ln 10 - \ln 12) + C \Rightarrow C = 0$

From (i), $f(4) = \frac{1}{2}\ln\left(\frac{17}{19}\right)$

5. Sol.(d)

(d) $\lim_{x\to0}\frac{e^{2|\sin x|}-2|\sin x|-1}{x^2}$

$\lim_{x\to0}\frac{e^{2|\sin x|}-2|\sin x|-1}{|\sin x|^2}\times\frac{\sin^2 x}{x^2}$

Let $|\sin x|=t$

$\lim_{t\to0}\frac{e^{2t}-2t-1}{t^2}\times\lim_{x\to0}\frac{\sin^2 x}{x^2}=\lim_{t\to0}\frac{2e^{2t}-2}{2t}\times1$

Using L' HOPITAL Rule $\lim_{t\to0}\frac{4e^{2t}}{2}=2$

6. Sol.(a)

Given that

$$\int_{\alpha}^{\log_e 4}\frac{dx}{\sqrt{e^x-1}}=\frac{\pi}{6}$$

Let $e^x-1=t^2\Rightarrow\int\frac{2dt}{t^2+1}$

$=2\tan^{-1}t=2\tan^{-1}\left(\sqrt{e^x-1}\right)\Big|_{\alpha}^{\log_e 4}$

$=2\left[\tan^{-1}\sqrt{3}-\tan^{-1}\sqrt{e^\alpha-1}\right]=\frac{\pi}{6}$

$=\frac{\pi}{3}-\tan^{-1}\sqrt{e^\alpha-1}=\frac{\pi}{12}\Rightarrow\tan^{-1}\sqrt{e^\alpha-1}=\frac{\pi}{4}$

$\therefore e^\alpha=2,\ e^{-\alpha}=\frac{1}{2}$

$x^2-\left(2+\frac{1}{2}\right)x+1=0\Rightarrow2x^2-5x+2=0$

7. Sol.(c)

Since,

$$\lim_{x\to2p^+}\left(\frac{1-\cos(x^2-4px+q^2+8q+16)}{(x^2-4px+q^2+8q+16)^2}\right)\left(\frac{(x^2-4px+q^2+8q+16)^2}{(x-2p)^4}\right)$$

$$\lim_{h\to0}\frac{1}{2}\left(\frac{(2p+h)^2-4p(2p+h)+q^2+8q+16}{h^2}\right)^2=\frac{1}{2}$$

(Using L' Hospital's)

So. $\lim_{x\to2p^+}[f(x)]=0$

8. Sol.(d)

$\because f(x)$ has extremum values at $x=1$ and $x=2$

$\because f'(1)=0$ and $f'(2)=0$

As, $f(x)$ is a polynomial of degree 4.

Suppose $f(x)=Ax^4+Bx^3+Cx^2+Dx+E$

$\because\ \lim_{x\to0}\left(\frac{f(x)}{x^2}+1\right)=3$

$\Rightarrow\ \lim_{x\to0}\left(\frac{Ax^4+Bx^3+Cx^2+Dx+E}{x^2}+1\right)=3$

$\Rightarrow\ \lim_{x\to0}\left(Ax^2+Bx+C+\frac{D}{x}+\frac{E}{x^2}+1\right)=3$

As limit has finite value, so $D=0$ and $E=0$

Now $A(0)^2+B(0)+C+0+0+1=3$

$\Rightarrow c+1=3\Rightarrow c=2$

$f'(x)=4Ax^3+3Bx^2+2Cx+D$

$f'(1)=0\Rightarrow\ 4A(1)+3B(1)+2C(1)+D=0$

$\Rightarrow\ 4A+3B=-4$

$f'(2)=0\Rightarrow4A(8)+3B(4)+2C(2)+D=0$

$\Rightarrow\ 8A+3B=-2$

From equations (i) and (ii), we get

$A=\frac{1}{2}$ and $B=-2$

So, $f(x)=\frac{x^4}{2}-2x^3+2x^2$

Therefore, $f(-1)=\frac{(-1)^4}{2}-2(-1)^3+2(-1)^2$

$=\frac{1}{2}+2+2=\frac{9}{2}$. Hence $f(-1)=\frac{9}{2}$

9. Sol.(a)

Let I

$$=\int\frac{\sin^2x\cos^2x}{(\sin^5x+\cos^3x\sin^2x+\sin^3x\cos^2x+\cos^5x)^2}dx$$

$$=\int\frac{\sin^2x\cos^2x}{(\sin^3x+\cos^3x)^2}dx=\int\frac{\tan^2x\cdot\sec^2x}{(1+\tan^3x)^2}dx$$

Now, put $(1+\tan^3x)=t$

$\Rightarrow\ 3\tan^2x\sec^2x\,dx=dt$

$\therefore\ I=\frac{1}{3}\int\frac{dt}{t^2}=-\frac{1}{3t}+C=\frac{-1}{3(1+\tan^3x)}+C$

10. Sol.(b)

Let $g(x)=ax+b$

Now function f(x) is continuous at x = 0

$\therefore \lim_{x\to 0^+} f(x)=f(0)$

$\Rightarrow \lim_{x\to 0}\left(\frac{1+x}{2+x}\right)^{\frac{1}{x}}=g(0)=b \Rightarrow 0=b$

$\therefore g(x)=ax$

Now, for x > 0

$f'(x)=\frac{1}{x}\cdot\left(\frac{1+x}{2+x}\right)^{\frac{1}{x}-1}\cdot\frac{1}{(2+x)^2}+\left(\frac{1+x}{2+x}\right)^{\frac{1}{x}}\cdot\ln\left(\frac{1+x}{2+x}\right)\left(-\frac{1}{x^2}\right)$

$\therefore f'(1)=\frac{1}{9}-\frac{2}{3}\cdot\ln\left(\frac{2}{3}\right)$

And $f(-1)=g(-1)=-a$

$\therefore a=\frac{2}{3}\ln\left(\frac{2}{3}\right)-\frac{1}{9}$

$\therefore g(3)=2\ln\left(\frac{2}{3}\right)-\frac{1}{3}=\ln\left(\frac{4}{9}\right)-\ln e^{1/3}=\ln\left(\frac{4}{9e^{1/3}}\right)$

11. Sol.(b)

Let $I=\int_{-1}^{+1}\frac{\cos\alpha x}{1+3^x}dx$ ...(i)

$I=\int_{-1}^{+1}\frac{\cos\alpha x}{1+3^{-x}}dx$ ...(ii)

$\left(\text{Using }\int_a^b f(x)dx=\int_a^b f(a+b-x)dx\right)$

Add (i) and (ii)

$2I=\int_{-1}^{+1}\cos(\alpha x)dx=2\int_0^1\cos(\alpha x)dx$

($\because g(x)=\cos\alpha x$ is even function)

$I=\frac{\sin\alpha}{\alpha}=\frac{2}{\pi}$ (given); $\therefore \alpha=\frac{\pi}{2}$

12. Sol.(d)

Given, $f(x)=[x(x-1)]+\{x\}$

$f(0^+)=-1+0=-1,\quad f(1^+)=0+0=0$

$f(0)=0,\quad f(1)=0$

$f(0^-)=0+1=1\quad f(1^-)=-1+1=0$

$\therefore$ f(x) is continuous at x = 1, discontinuous at x = 0

13. Sol.(b)

Consider $\lim_{x\to 0}\left(\frac{x-\sin x}{x}\right)\sin\left(\frac{1}{x}\right)$

$=\lim_{x\to 0}\left[1-\frac{\sin x}{x}\right]\times\lim_{x\to 0}\sin\left(\frac{1}{x}\right)$

$=\left[1-\lim_{x\to 0}\frac{\sin x}{x}\right]\times\lim_{x\to 0}\sin\left(\frac{1}{x}\right)=0\times\lim_{x\to 0}\sin\left(\frac{1}{x}\right)=0$

14. Sol.(b)

$\lim_{x\to 0}f(x)=a\ell n2\ell n3=f(0)$

$\lim_{x\to 0}\frac{72^x-9^x-8^x+1}{\sqrt{2}-\sqrt{1+\cos x}}=\lim_{x\to 0}\frac{(8^x-1)(9^x-1)}{\sqrt{2}-\sqrt{1+\cos x}}$

$\lim_{x\to 0}\left(\frac{8^x-1}{x}\right)\left(\frac{9^x-1}{x}\right)\left(\frac{x^2}{1-\cos x}\right)(\sqrt{2}+\sqrt{1+\cos x})$

$\therefore ln8\times ln9\times 2\times\sqrt{2}=24\sqrt{2}\,ln2\,ln3$

$\therefore a=24\sqrt{2},\ a^2=576\times 2=1152$

15. Sol.(d)

$g(x)=h(e^x).e^{h(x)}$

Differentiating both sides

$g'(x)=h(e^x).e^{h(x)}.h'(x)+e^{h(x)}h'(e^x).e^x$

$g'(0)=h(1)e^{h(0)}h'(0)+e^{h(0)}h'(1)$

$=2+2=4$

16. Sol.(a)

$$g(\alpha)=\int_{\pi/6}^{\pi/3}\frac{\sin^\alpha x}{\cos^\alpha x+\sin^\alpha x}dx \quad \ldots(1)$$

$$=\int_{\pi/6}^{\pi/3}\frac{\sin^\alpha\left(\frac{\pi}{2}-x\right)}{\cos^\alpha\left(\frac{\pi}{2}-x\right)x+\sin^\alpha\left(\frac{\pi}{2}-x\right)}dx$$

$$=\int_{\pi/6}^{\pi/3}\frac{\cos^\alpha x}{\sin^\alpha x+\cos^\alpha x}dx \quad \ldots(2)$$

From (1) + (2),

$$2.g(\alpha)=\int_{\pi/6}^{\pi/3}\frac{\sin^\alpha x+\cos^\alpha x}{\sin^\alpha x+\cos^\alpha x}dx=\int_{\pi/6}^{\pi/3}dx=\frac{\pi}{3}-\frac{\pi}{6}=\frac{\pi}{6}$$

$g(\alpha)=\frac{\pi}{12}$ $\Rightarrow g(\alpha)$ is a constant function and hence an even function.

17. Sol.(b)

Let, $\ell=\lim_{x\to 0^+}(\sin(\sin kx)+\cos x+x)^{\frac{2}{x}}=e^6$

Taking log on both sides,

$\Rightarrow \ln l=\lim_{x\to 0^+}\frac{2}{x}(\sin(\sin kx)+\cos x+x-1)$

$\Rightarrow \ln l=\lim_{x\to 0^+}2\left(\frac{\sin(\sin kx)}{\sin kx}\cdot\frac{\sin kx}{kx}\cdot\frac{kx}{x}+1-\frac{(1-\cos x)}{x^2}.x\right)$

$\Rightarrow \ln\ell=2(k+1)\Rightarrow l=e^{2(k+1)}=e^6$

$k+1=3\Rightarrow k=2$

18. Sol.(c)

Given, $\frac{dT}{dt} = -k(T-80)$

$$\Rightarrow \int_{160}^{T} \frac{dT}{(T-80)} = \int_{0}^{t} -Kdt \Rightarrow [\ln|T-80|]_{160}^{T} = -kt$$

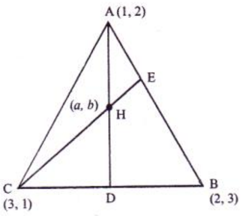
$$\Rightarrow \ln\left|\frac{T-80}{80}\right| = -kt \Rightarrow T = 80 + 80e^{-kt}$$

$$120 = 80 + 80e^{-k.15} \Rightarrow e^{-k15} = \frac{1}{2}$$

$$\therefore \quad T(45) = 80 + 80e^{-k.45}$$

$$= 80 + 80\,(e^{-k.15})^3 = 80 + 80 \times \frac{1}{8} = 90$$

19. Sol.(a)



Slope of AB = 1,
then scope of CE = – 1
$\therefore$ Equation of CE
$y - 1 = -(x - 3)$
$x + y = 4$
Since, orthocentre lies on the line $x + y = 4$
So, $a + b = 4$

$$I_1 = \int_a^b x \sin(x(4-x))dx \qquad \text{... (i)}$$

Using king rule

$$I_1 = \int_a^b (4-x)\sin g(x(4-x))dx \qquad \text{... (ii)}$$

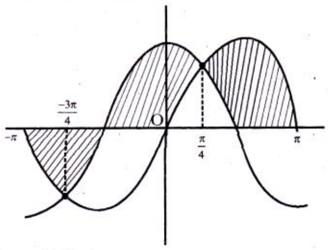
Adding equation (i) and (ii)

$$2I_1 = \int_a^b 4\sin g(x(4-x))\,dx \Rightarrow 2I_1 = 4I_2 \Rightarrow \frac{I_1}{I_2} = 2$$

$$\Rightarrow \frac{36I_1}{I_2} = 72$$

20. Sol.(d)

Since graphs of the given function are



So, required

$$\text{Area} = \left|\int_{-\pi}^{\frac{-3\pi}{4}} \sin x dx\right| + \left|\int_{\frac{-3\pi}{4}}^{\frac{-\pi}{2}} \cos x dx\right| + \int_{\frac{-\pi}{2}}^{\frac{\pi}{4}} \cos x dx + \int_{\frac{\pi}{4}}^{\pi} \sin x dx = 4$$

Integer Type

1. Sol.(62)

Area define by

$$A = \int_{-1}^{0} (x^2 - 3x)dx + \int_{0}^{2} (3x - x^2)dx$$

$$A = -\left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_0^{-1} + \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^2$$

$$A = \frac{1}{3} + \frac{3}{2} + \frac{12}{2} - \frac{8}{3} \Rightarrow A = \frac{15}{2} - \frac{7}{3} = \frac{31}{6}$$

2. Sol.(2)

$$\lim_{\alpha \to 0} \frac{e^{\cos\alpha^n} - e}{\alpha^m} = \frac{-e}{2}$$

$$\Rightarrow \lim_{\alpha \to 0} \frac{e\left[e^{\cos\alpha^n - 1} - 1\right]}{\cos\alpha^n - 1} \times \frac{\cos\alpha^n - 1}{\alpha^m} = \frac{-e}{2}$$

$$\Rightarrow e \lim_{\alpha \to 0} \frac{-2\sin^2 \frac{\alpha^n}{2}}{\left(\frac{\alpha^n}{2}\right)^2} \times \frac{\left(\frac{\alpha^n}{2}\right)^2}{\alpha^m} = \frac{-e}{2}$$

$$\Rightarrow \frac{-e}{2}\alpha^{2n-m} = \frac{-e}{2} \Rightarrow \frac{m}{n} = 2$$

3. Sol.(4)

$$I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{dx}{\left(1+e^{\sin x}\right)(2-\cos 2x)} \qquad \text{...(i)}$$

$$I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{dx}{\left(1+e^{-\sin x}\right)(2-\cos 2x)}$$

$$\left[\text{using} \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$$

$$I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{e^{\sin x}}{\left(e^{\sin x}+1\right)(2-\cos 2x)} \qquad \text{...(ii)}$$

Adding (i) and (ii):

$$2I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{1+e^{\sin x}}{\left(e^{\sin x}+1\right)(2-\cos 2x)}dx$$

$$= \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{1}{2-\cos 2x}dx$$

$$I = \frac{2}{\pi}\int_0^{\pi/4} \frac{dx}{1+2\sin^2 x}$$

$$= \frac{2}{\pi}\int_0^{\pi/4} \frac{\sec^2 x}{\sec^2 x + 2\tan^2 x}dx = \frac{2}{\pi}\int_0^{\pi/4} \frac{\sec^2 x}{1+3\tan^2 x}dx$$

Put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

At $x = 0$, $t = 0$; At $x = \frac{\pi}{4}$, $t = 1$

$$\therefore I = \frac{2}{\pi}\int_0^1 \frac{1}{1+3t^2}dt = \frac{2}{\pi}\left[\frac{1}{\sqrt{3}}\tan^{-1}\left(\sqrt{3}t\right)\right]_0^1$$

$$= \frac{2}{\pi}\left[\frac{1}{\sqrt{3}} \times \frac{\pi}{3}\right] = \frac{2}{3\sqrt{3}}$$

$$\therefore 27I^2 = 4$$

4. Sol.(309)

$$g(x) = h(e^x).e^{h(x)}$$

Differentiating both sides

$g'(x) = h(e^x).e^{h(x)}.h'(x) + e^{h(x)}h'(e^x).e^x$

$g'(0) = h(1)e^{h(0)}h'(0) + e^{h(0)}h'(1)$

$= 2 + 2 = 4$

$\Rightarrow \quad m = \frac{103}{32}$ So $\frac{8f'(8)}{f'\left(\frac{1}{8}\right)} = 8 \times \frac{2mx|_{x=8}}{6x + \frac{k}{2\sqrt{x+1}}\Big|_{x=\frac{1}{8}}}$

$= \frac{8 \times 2 \times 8 \times \frac{103}{32}}{\frac{16}{12}} = 309$

5. Sol.(7)

$$\int \frac{1}{\sqrt[5]{(x-1)^4(x+3)^6}}\,dx = A\left(\frac{\alpha x - 1}{\beta x + 3}\right)^B + C$$

Let $I = \int \frac{1}{(x-1)^{4/5}(x+3)^{6/5}}\,dx$

$I = \int \frac{1}{\left(\frac{x-1}{x+3}\right)^{4/5}(x+3)^2}\,dx$

Let $\left(\frac{x-1}{x+3}\right) = t \Rightarrow \frac{4}{(x+3)^2}dx = dt$

$\Rightarrow dx = \frac{(x+3)^2}{4}dt \Rightarrow I = \frac{1}{4}\int \frac{1}{t^{4/5}}dt = \frac{1}{4}\frac{t^{1/5}}{1/5} + c$

$I = \frac{5}{4}\left(\frac{x-1}{x+3}\right)^{1/5} + C$

$\therefore \quad A = \frac{5}{4}, \quad \alpha = \beta = 1 \quad B = \frac{1}{5}$

$\therefore \quad \alpha + \beta + 20AB = 2 + 20 \times \frac{5}{4} \times \frac{1}{5} = 7$